Design Example – Analysis and Design of Jib Crane Boom

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Introduction
This article provides an example of how to design the boom of a jib crane to NZS 3404 (SNZ, 2007). This example illustrates the use of second order and restraint requirements of NZS 3404.

Description
The jib crane being analysed is shown in Figure 1. The drawing shows key points on the boom; A, B, C and D and the bending moment diagram and reactions along the boom and in the hanger for a downwards force P at the end of the boom.

The position of the points is as follows:
A is where the load is lifted
B is at the point of hanger attachment to the boom
C is at the swivel pin
D is where the cantilever supporting the swivel pin intersects with the column

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Bending Moment Determination
The bending moment determined in Figure 1 is derived from first principles for this structure. The bending moment in the supporting column is not shown because this is dependent on the interaction of the action effects from the crane boom with those from the building frame and the latter are not known.

Points to note in the actions shown are:
- the step in bending moment at B shown in Figure 1 is due to the local eccentricity generated at the connection by the transfer of forces shown in Figure 2
- the point C is a flexural pin hence zero moment at that point

The bending moment generated in the boom by the hanger is determined by resolving the hanger forces into the horizontal and vertical components at the point of intersection of the hanger centreline with the boom top flange. This is the location of point B, as shown in Figure 2. The moment is the magnitude of the horizontal force acting about a lever arm of half the boom depth as shown in Figure 2. For this case the moment = 0.39\(P\), which means the bending moment diagram along the boom has a step in it of that magnitude at location B.

Second Order Effects and Elastic Stability
Figure 1 shows the first order bending moments in the crane boom. Second order effects in accordance with NZS 3404 section 4.4.2 need to be considered, especially in column BC, although it will be straightforward to show when applying clause 4.4.3.2.2 that they don’t magnify the first order design actions. Elastic stability of column BC about the minor axis direction should also be considered by checking that the elastic buckling load factor for that member about the minor axis \(\lambda_{cy}\) ≥ 3.5 as required by NZS 3404 Clause 4.9.1 when a second order analysis is not performed directly. \(\lambda_{cy} = N_{umby}/N^*\) where \(N_{umby} = (\pi^2 EI_y)/(k_e L_{BC})^2\), \(N^* = 3.9P\) and \(k_e = 1.0\).

Analysis and Design
At points A, B, C and D the restraint provided to the boom cross section is determined in accordance with NZS 3404 clause 5.4.2.

The member segment AB is unrestrained (U) at end A as a minimum consideration. If the load is applied from the bottom flange the restraint will be slightly better than (U) but if the load is applied at or above the shear centre then U is appropriate. The member segment AB has partial (P) restraint at end B due to the stabilising effect of the hanger load being applied at the top on the opposite side of the section to the critical flange. See HERA Design & Construction Bulletin No 41 (Clifton, 1998) pages 7 to 9 for the background to this. See also sections 5.4.2.2 and 5.4.2.3 of (Clifton, 1994) for further explanation. This segment is a beam subject to cantilever induced negative bending moment and no axial load.

To ensure that the bottom flange at B has effective lateral restraint provided by the web back to the point of attachment of the hanger, which is required for the (P) cross section restraint at point B, the out of plane moment capacity of the web to resist a restraint force equal to 2.5% of the axial force in the bottom flange generated by the cantilever bending moment at B from segment AB must be determined. This restraint force
acts out of the plane of the cross section at location B. The effective length of web that participates in resisting this action is the length \(b_s\) shown in Figure 2 plus 5x the flange thickness plus 2x the clear depth of web. How to carry out this calculation is described in section 2.2.2 of HERA Report R4-92 (Clifton, 1997).

Member segment BC has (P) restraint at end B and can be considered to have partial (P) restraint at end C due to the cantilever segment CD. It is subject to the major axis bending moment shown, a shear force of P/3.9 and an axial load of 3.9P. The effective length for bending, \(L_{ez}\), is determined from Clause 5.6 of NZS 3404 for the cross section restraint condition (PP) and the loads applied at the segment ends. The loads can be considered applied through the shear centre.

Member BC is a beam column. The effective lengths for axial compression can be taken as 1.0 x the length \(L_{BC}\) for second order effect determination (see above) and for member design, \(\phi N_{cy}\) will be the critical member compression capacity. If the segment has full lateral restraint then combined actions will involve \(\phi M_s\) and \(\phi N_{cx}\), if not then \(\phi M_{bx}\) and \(\phi N_{cy}\).

Member segment CD is also a beam column. It should be designed as a cantilever from the supporting column resisting the following:
- the bending moment generated by P/3.9 acting upwards
- the axial load 3.9P acting along the member
- \(M^*_{y} = L_{CD} \times 0.025 \times 3.9P\) acting transversely on the swivel pin at C back to the base of the member at D.
- for major axis bending this member can be considered to be (U) at end C and F restraint at end D
- the critical location will be at the base of the member at D
- the connection into the column must be designed to resist these actions

The supporting column must resist all these actions including the torsion from member CD

The hanger force is 4.1P. The cantilever supporting the hanger off the main supporting column must be designed for this. Figure 2 shows a suitable concept used for the hanger to boom connection.

**Worked Example**

Consider boom is 200UC60 and the factored load \(P^*\) is 50 kN. All clause references are to NZS 3404 (SNZ,2007). (ASI, 1999) have been used for section capacity values. Only checks for member segments AB and BC are included in this example.

**Member Segment AB**

Design moment at point B:
\[M^*_B = P^* L = 50 \times 1.1 = 55 \text{kNm}\]

Rapid estimate of \(\phi M_{bx}\) using clause 5.6.2 (A more accurate approach is to use Appendix H as described in DCB 41.):
\[L_e = k_t L = 1.08 \times 1.1 = 1.19 \text{m}\]
\[\psi = 1.0, k_t = 1.0\]
\[\alpha_m = 1.25 \text{ from table 5.6.2}\]
\[\phi M_{bx} = \alpha_m \psi \phi M_{bx} \leq M_{bx}\]
\[\alpha_m \psi > 1\]
\[\phi M_{bx} = 177 \text{kNm} > M^*_B \text{ OK!}\]

Check restraint at B using approach in HERA Report R4-92
Restraining force:
\[R^*_r = 0.025 \times \frac{M^*_r}{\psi - t_f} = 0.025 \times \frac{55}{0.21 - 0.0142} = 7 \text{kN}\]

Design moment to be resisted by the unstiffened web:
\[M_{nw} = R^*_r \times (1.5t_f)^2 = 7 \times (1.5 \times 0.0142)^2 = 1.32 \text{kNm}\]
Design moment capacity of the unstiffened web:

\[ L_w = b_w + 5t_w + 2(d - 2t) = 300 + 5 \times 14.2 + 2 \left( 10 - 2 \times 14.2 \right) = 734 \text{ mm} \]

\[ Z_{sw} = 0.25L_w t_w^2 = 0.25 \times 734 \times 9.3^2 = 15,875 \text{ mm}^3 \]

\[ \phi M_{yw} = 0.9 Z_{sw} f_{yw} = 0.9 \times 15,875 \times 320 \times 10^{-6} = 4.57 \text{ kNm} > M_{yw} \text{ OK!} \]

Design shear capacity:

\[ \phi V_v = 337 \text{ kN} \]

\[ V^* = P^* = 50 \text{ kN} < \phi V_v \text{ OK!} \]

**Member Segment BC**

Check to see if second order effects may be neglected for bending moments in accordance with clause 4.4.2.2.1:

\[ N^* = 3.9 P^* = 3.9 \times 50 = 195 \text{ kN} \]

\[ N_{ombx} = \frac{\phi EI_x}{L_{bc}} = \frac{\pi^2 \times 200 \times 61.3}{40 \times 3.9^2} = 7,955 \text{ kN} \]

\[ \lambda_{cx} \geq \frac{N_{ombx}}{N^*} = \frac{7,955}{195} = 41 > 10 \]

Hence second order effect may be neglected in accordance with clause 4.4.2.2.1. However for completeness if second order effects are considered than according to clause 4.4.3.2.2 the moment amplification factor can be shown to equal 1.

\[ \delta_x = \frac{c_m}{1 - \frac{N^*}{N_{ombx}}} = \frac{0.6}{1 - \frac{195}{7,955}} = 0.62 < 1.0 \therefore \delta_x = 1.0 \]

Check elastic stability of column BC about the minor axis direction:

\[ N_{omby} = \frac{\phi EI_y}{L_{bc}} = \frac{\pi^2 \times 200 \times 20.4}{40 \times 3.9^2} = 2,647 \text{ kN} \]

\[ \lambda_{cy} \geq \frac{N_{omby}}{N^*} = \frac{2,647}{195} = 13.5 \geq 3.5 \text{ OK!} \]

Design minor axis axial capacity:

\[ L_e = k_e L_{bc} = 1.0 \times 3.9 = 3.9 \text{ m} \]

\[ \phi N_{cy} = 1359 \text{ kN} > 195 \text{ kN} \text{ OK!} \]

Design major axis moment capacity:

\[ \alpha_m = 1.75 \text{ from table 5.6.1} \]

Using table 5.6.3(1) for (PP)

\[ k_i = 1 + \left[ 2 \left( \frac{d}{L} \right) \left( \frac{t_i}{2t_w} \right) \right] 1 + \left[ 2 \left( \frac{210}{3900} \right) \left( \frac{14.2}{2 \times 9.3} \right) \right] 1 = 1.05 \]

\[ L_e = k_i L = 1.05 \times 3.9 = 4.09 \text{ m} \]

\[ \zeta_t = 1.0, k_i = 1.0 \]

Using (ASI,1999)

\[ \alpha_s \phi M_{sx} = 139 \text{ kNm} \]

\[ \alpha_s = \frac{139}{177} = 0.79 \]

\[ \alpha_m \alpha_s = 1.75 \times 0.79 = 1.38 > 1 \therefore \text{ Full lateral restratint} \]

\[ \phi M_{sx} = 177 \text{ kNm} \]

\[ M^* = M^*_s - 0.39 P^* = 55 - 0.39 \times 50 = 35.5 \text{ kNm} < \phi M_{sx} \text{ OK!} \]
Combined actions:

\[ \phi M_{x} = 1.18 \frac{M_{x}}{\phi N_{x}} \left( 1 - \frac{N^*}{\phi N_{x}} \right) = 1.18 \times 177 \left( 1 - \frac{195}{2060} \right) = 189 > 177 \therefore \phi M_{x} = 177 \text{kNm} > M^* \text{ OK!} \]

\[ \phi M_{x} = \frac{\phi M_{x}}{\phi N_{x}} \left( 1 - \frac{N^*}{\phi N_{x}} \right) = 177 \left( 1 - \frac{195}{1790} \right) = 158 \text{kNm} > M^* \text{ OK!} \]

References

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