Design Example – Continuous Column in Simple Construction Effective Length

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Introduction
For continuous columns in simple construction the restraint provide by the continuity of the column past a floor level may be used to reduce the column effective buckling length. Generally the column effective buckling length is taken as the interstorey height i.e. $k_e = 1.0$. This is conservative and in some situations, such as adding additional floors to a building, the extra loads on the existing column may exceed the calculated axial capacity. In this situations a more detailed calculations of the effective length can be carried out such that $k_e < 1.0$, resulting in an increased computed axial capacity. A worked example is used to illustrate this. The worked example uses the same column configuration as in Steel Advisor MEM8001 (Cowie, Fussell, 2012).

The more accurate procedure for determining the effective length involves the use of NZS 3404 Appendix G (SNZ, 2007). This approach is also used in determining accurately the effective length for truss members and is described in section 7.6.1 of HERA Report R4-80 Structural Steelwork Limit State Design Guide (Clifton, 1994).

![Figure 1: Column details for worked example](image-url)
**Design Parameters**

The column shown in figure 1 is a 200UC46 G300. The column is continuous and forms part of a structure of simple construction. The column is nominally pinned at the base. Beams are connected to the column flange by flexible end plates. The column splice connections are located away from floor levels and have moment capacity. The minimum moment capacity of such column splices is stated in clause 9.1.4 of NZS 3404 (SNZ 1997). Compression load in the column between levels 1 and 2 is 377 kN. The exercise is to calculate the effective length of the column member from ground to level 1.

**Effective Length**

Appendix G of NZS 3404 (SNZ, 2007) is used to determine the stiffness ratios at each end of the column member. The effective length is then read off figure 4.8.3.3(a) of NZS 3404.

For the column member ground to level 1, restraint is provided at level 1 due to the 'backspan' of the column from level 1 to level 2. Axial load in the backspan will reduce the effective stiffness provided. No restraint is provided by the beams at level 1 as the beam to column connections are shear only and are free to rotate.

The nominal pin base connection will also provide restraint to the column member at the ground level. A fixed base will provide increased restraint. NZS 3404 4.8.3.4.1 states appropriate stiffness ratios for nominal pin and fixed base connections. The stiffness ratio (γ) limit is not less than 10 for nominal pin base and not less than 0.6 for a fixed base.

The equation in appendix G of NZS 3404 to determine the stiffness ratio provided at each end of the column member is as follows.

\[ \gamma = \frac{1}{L} \sum \left( \beta_e \alpha_{sr} \frac{1}{L} \right) \]  
(1)

where:

\[ \frac{1}{L} = \text{stiffness in the plane of bending of the column, ground to level 1} \]

\[ \sum \left( \beta_e \alpha_{sr} \frac{1}{L} \right) = \text{summation of the stiffness in the plane of bending of all the restraining members rigidly connected at the end to the member under consideration} \]

\[ \beta_e = \text{modification factor given in table 4.8.3.4 NZS 3404 to account for the end conditions at the far end of the column restraining member} \]

\[ \alpha_{sr} = \text{theoretical stability function multiplier, or the approximation shown in figure G1 NZS 3404 to account for the effect of the design axial force (} N_i ^* \text{) in the column restraining member on its flexural stiffness.} \]

To use figure G1 NZS 3404 to determine \( \alpha_{sr} \), the value of \( \rho \) must be first calculated as follows

\[ \rho = \frac{N_i ^*}{N_{oLr}} \]  
(2)

where:

\[ N_{oLr} = \frac{\pi^4 E L}{L^2} \]  
(3)

The equation for stiffness ratio at each end of the column member is simplified as follows.

*Stiffness ratio for the end of the column member at Level 1*
The column is continuous and therefore the column backspan is rigidly connected to the column member. In accordance with table 4.8.3.4 NZS 3404 $\beta_{e} = 1$

The backspan will be in single curvature and so reduction in stiffness due to the presence of axial compression load using figure G1 of NZS 3404 is

$$\alpha_{b\text{(Level1- Level2)}} = 1 - \rho$$

The stiffness ratio of the column member at Level 1 can then be expressed as

$$\gamma_{\text{Level1}} = \frac{\frac{I_{(\text{Ground level1})}}{L_{(\text{Ground level1})}}}{\frac{1 - N_{\text{od,Level1- Level2}}}{N_{\text{od,Level1- Level2}}}} \times \frac{I_{(\text{Level1 Level2})}}{L_{(\text{Level1 Level2})}}$$

(4)

The stiffness ratio calculations are as follows

$$N_{\text{od,Level1- Level2}} = \frac{\pi^{2}EI_{(\text{Level1- Level2})}}{L_{(\text{Level1- Level2})}^{2}}$$

$$I_{(\text{Level1- Level2})} = I_{y} \text{ for a 200UC46} = 15.3 \times 10^{-6} \text{ m}^4$$

$$N_{\text{od,Level1- Level2}} = \frac{\pi^{2} \times 200 \times 10^{6} \times 15.3 \times 10^{-6}}{3^2} = 3,356 \text{ kN}$$

$$\gamma_{\text{Level1}} = \frac{\frac{15.3 \times 10^{-6}}{5}}{\frac{377}{3356} \times \frac{15.3 \times 10^{-6}}{3}} = 0.67$$

Note: In the example the minor axis I value was used ($I_{y}$). This is appropriate for calculating $k_{ex}$. By inspection buckling about the major axis will not be critical. In the unusual situation that a column had closely spaced restraint against minor axis buckling, major axis buckling may be the critical limit state and would need considering. In this instance $k_{ex}$ utilising $I_{x}$ could be calculated using the methodology just outlined $I_{y}$ to reduce $L_{ex}$.

**Stiffness ratio for the end of the column member at Ground**

The column is nominally pinned and 4.8.3.4.1(a) NZS 3404 is used to determine stiffness ratio.

$$\gamma_{\text{Ground}} = 10$$

**Effective length factor and length**

Having now determined the stiffness ratio at each end of the column member the effective length factor is determined using figure 4.8.3.3(a) NZS 3404

$$k_{e} = 0.83$$

The column member effective length is now determined using the effective length factor

$$L_{e} = k_{e}L = 0.83 \times 5 = 4.15 \text{ m}$$

NZS 3404 6.3.2

In Steel Advisor MEM8001 (Cowie, Fussell, 2012) a $k_{e}$ of 1.0 was used. By way of comparison, reducing the effective length to 4.15m from 5m resulted in an increase in column axial capacity from 786 kN to 989 kN (axial load capacities calculated using MEMDES v2.1 (NZ Steel, 2003))

**References**

Clifton, G. C., New Zealand Structural Steelwork Limit State Design Guides Volume 1, HERA Report R4-80, New Zealand Heavy Engineering Research Association, Manukau City, 1994

New Zealand Steel, MEMDES v2.1, November 2003