Deep Rafter Stability

Author: *Alistair Fussell*
Affiliation: *Steel Construction New Zealand Inc.*
Date: *17th March 2009*
Ref: *MEM3601*

Key Words
Tension side lateral restraint, elastic torsional restraints, deep rafters, elastic flexural torsional buckling analysis

Introduction

The New Zealand Steel Structures Standard (SNZ, 1997), in keeping with many international design standards has no minimum stiffness requirement for restraints systems preventing flexural-torsional buckling of steel sections bent about their major axis. To account for restraint system flexibility, NZS3404 categorises these systems as providing full or partial restraint. In the commentary to the Steel Structures Standard guidance is provided for roof rafters for the minimum ratio of purlin depth to rafter depth to ensure a lateral restraint system fully restrains the section. No such guidance is provided to allow designers to check that the restraining system is sufficiently rigid to provide at least partial restraint. This is particularly relevant for the bottom critical flanges of deep roof rafters restrained by a flexible tension side system in conjunction with full depth stiffeners (Figure 2). In this paper guidance is provided for structural engineers to help them address this situation.

Steel Structures Standard Restraint Provisions

Lateral restraints are provided to open steel (I and C) sections to increase their buckling capacity by restraining the critical compression flange. At a cross section that is considered to be fully, partially or laterally restrained against deflection, the restraint system is required to transfer 2.5 % of the critical flange force. No minimum stiffness value is given in the Steel Structures Standard (SNZ, 1997) even though the commentary acknowledges there may be situations where due to the large restraint force and the flexibility of the restraining system specific account must be taken of elastic rotational restraint stiffness. The reason for the omission of a minimum stiffness requirement follows the finding that the stiffness requirements are typically met by practical braces that satisfy the 2.5% strength rule.

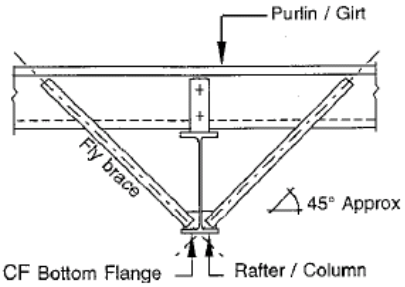

Figure 1: Roof Rafter Flybrace Detail (Clifton, 1997)

Deep Rafter Stability

A common detail for restraining the lower critical flanges of roof rafters is the use of a triangulated fly brace system (Figure 1).

There are some instances, particularly for architectural reasons, a moment connection between the purlins and the rafter top flange in conjunction with stiffeners is used to provide restraint to the bottom flange (Figure 2). This type of system is inherently more flexible than a fly brace system which relies on a triangulated system to transfer the bottom flange restraint force (0.025 x the compression flange force) to the purlins. In some large roof structures such as indoor or outdoor sports stadiums the rafter depth may exceed 2m.

When this type of detail is applied to a deep rafter with relatively flexible purlins the difficulty for the structural designer is that there is no quantitative guidance given to check the restraint system possesses adequate stiffness to be classified as a partial restraint. The structural implication of this lack of brace stiffness criteria is that in some instances

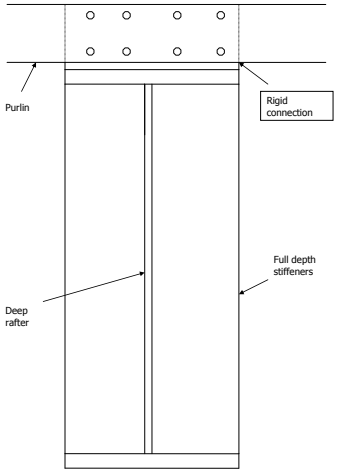

Figure 2: Deep Rafter with Flexible Tension Flange Restraint

the degree of restraint provided will be less than that assumed by the effective length provisions of the Steel Structures Standard. Underestimating the effective length will lead to an un-conservative assessments of the flexural capacity of a member. Before considering how this problem may be tackled, an overview will be given of the back ground to the Steel Structures Standard restraint provisions. This will give some insight into the role of restraint stiffness in the buckling behaviour of beams.

Background to the Steel Structures Standard Restraint Provisions

This background information has been taken from (Jeffers, 1992). To illustrate the role of the stiffness of the restraint system to the buckled shape of the critical compression flange of a beam, consider a brace with a central strut of stiffness k. The buckled shape of the strut is dependent on the brace stiffness. If the brace possesses adequate stiffness, a point of contraflexure will occur at this location (Figure 3 (c)), other- wise a bow shaped buckled profile forms (Figure 3 (b)). The minimum threshold stiffness to cause the points of contra-flexure to form at the brace position is k_i.

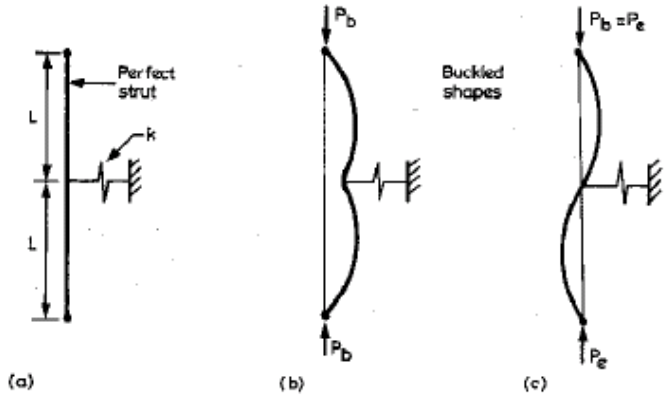

Figure 3: Strut With a Central Brace (Jeffers, 1992)

A plot of the strut buckling load P_b normalised with the strut load at Euler buckling P_e, shows the buckling load increases with brace stiffness until the threshold stiffness k_i (Figure 4). Above this value of stiffness the strut buckling load remains constant at the Euler buckling load P_e .

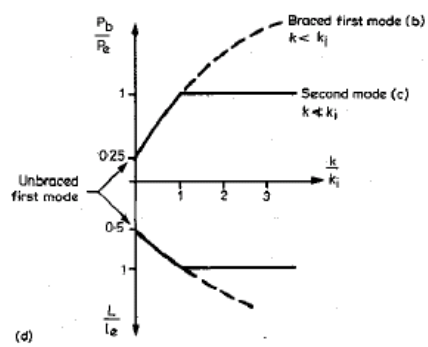

Figure 4: Strut Capacity Related to Brace Stiffness (Jeffers, 1992)

Figure 5 shows the relationship between the brace stiffness and the force required to be carried by the brace. Initially the brace force increases with increase in brace stiffness. The maximum required brace force is associated with a stiffness corresponding to the critical stiffness value k_i. If there is even the slightest geometric imperfection in the strut an infinite force is required to prevent buckling at this value of stiffness. The brace force then reduces with increasing stiffness.

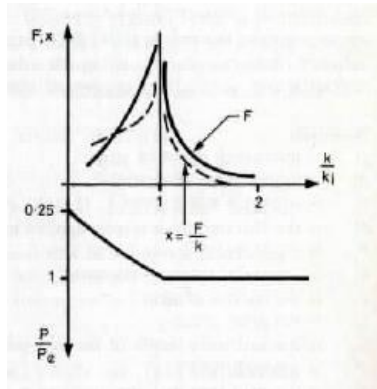

Figure 5: Brace Force Relationship to Brace Stiffness (Jeffers, 1992)

The relationship between the brace stiffness (k), brace displacement (x) and brace force (F) for $P=P_e$ is shown in Table 1

k	x	F
k_i	Infinite	Infinite, collapse
$1.25k_i$	$4x_0$	$P_e/25$
$1.5k_i$	$2x_0$	$P_e/40$
$2k_i$	X_0	$P_e/60$
$3k_i$	$0.5x_0$	$P_e/80$
$5k_i$	$0.25x_o$	$P_e/100$
infinite	0	$P_e/125$ not possible

Table 1: Relation Between Brace Stiffness (k), Brace Displacement (x), and Brace Force (F) for $P=P_e$ (Jeffers, 1992)

The minimum threshold stiffness is dependent on the number of braces. A greater brace stiffness is required to cause the point of contra-flexure to form at the restraints as the number of braces increases (Table 1).

No. of braces	1	2	3	4	infinite
$k_i=P_e/L$ times	2	3	3.41	3.63	4

Table 2: Variation in k_i with the number of regularly spaced braces (Jeffers, 1992)

In the now superceded Australian Steel Structures Standard AS 1250-1981, a minimum brace stiffness of 10P/L $=2.5k_i$ was specified along with a maximum displacement of 0.0025L. The minimum brace force was set at P/40 where P was the compression force in the critical flange at location being restrained.

Flexural Torsional Buckling

When a typical open structural section is subject to bending about its major axis, it is prone to buckling out of plane unless adequate restraint is provided to prevent this mode of beam failure (Figure 6). This buckling may occur at a moment significantly less than the members in plane section capacity.

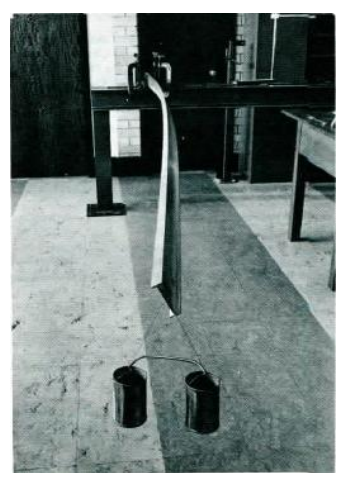

Figure 6: Flexural Torsional Buckling (from Trahir and Bradford, 1988)

This type of failure is known as flexural-torsional buckling because at the critical applied moment M_{cr}, the section fails when the beam critical flange (compression flange) begins to displace laterally due to the twisting of the section. Open I sections are particularly susceptible to this type of failure due to their lack of minor axis and torsional stiffness.

The Steel Structures Standard approach to determining the bending capacity of a beam subject to flexure is to introduce a slenderness factor α_s which reduces the section capacity for flexural-torsional buckling.

$$\phi M_b = \phi \alpha_m \alpha_s M_s$$

The section capacity is related a reference buckling moment by the following relationship which accounts for real beam behaviour (residual stresses and

geometric imperfections).

$$\alpha_s = 0.6\left\{\sqrt{\left[\left(\frac{M_{sx}}{M_{oa}}\right)+3\right]}-\left(\frac{M_{sx}}{M_{oa}}\right)\right\}$$

The Steel Structures Standard (SNZ, 1997) reference buckling moment M_o is determined for a particular beam span arrangement, support conditions and moment gradient. In this case a simply supported beam, with rigid twist restraints but free to rotate in plane at the supports and subject to a uniform moment. Since elastic flexural –torsional buckling is influenced by many factors including, the beam geometry, the distribution of loading on it and the effect of end and intermediate lateral restraints, an effective length concept has been introduced that allows for a variation in support and loading conditions from the standard reference beam. The Steel Structures Standard (SNZ, 1997) also provides a moment magnifier factor α_m to account for a non-uniform moment distribution.

Effective Length Concept

Jeffers (1992) defined the effective length as:

This is the length of an idealised member that can sustain the same buckling load as the actual braced member (of the same cross section) in the structure.

The effective length in NZS 3404 is of the following form:

$$L_e = k_r k_t k_l$$

where:

k_r lateral rotation restraint factor
k_t twist restraint factor
k_l load height factor

To determine k_t from the Steel Structures Standard, the designer must classify the lateral restraint system as providing the cross section with full, partial, lateral or rotational restraint. Guidance is provided in various publications for classifying restraint systems typically encountered in steel building construction (Clifton, 1997).

From the discussion on the background to the Steel Structures Standard restraint provisions, it should be apparent there is structural significance to the stiffness of the restraint system. If for the case of deep rafters with flexible tension side restraints, the restraints are incorrectly designated as partial restraints, there will be a departure from the assumed buckled shape and therefore a reduction in the compressive force in the critical flange when flange instability occurs.

Deep Rafters with Flexible Tension Side Restraints

As discussed in the previous section, before determining the flexural capacity of a deep rafter, the structural designer needs to designate the degree of restraint provided by the lateral restraint system. This is either fully or partially effective in restraining the compression flange. To check if the restraint is fully effective the stiffness provisions of AS 1250 may be used. If the lateral restraint system does not met this stiffness criteria, and the structural engineer is unsure whether their proposed flexible tension side restraint system has sufficient stiffness to be designated a partial restraint, they can refer to the U frame provisions of the Australian Bridge Standard AS 5100.6 Steel and Composite Construction.

These provisions allow an effective length based on the elastic stiffness of the lateral restraints to be computed. This method is typically used for through deck bridge structures (Figure 7). Here restraint is provided to the critical top flange by a series of frames. This configuration has similarities to deep rafter bottom flanges restrained by a tension flange restraint system. In this situation in-plane roof bracing laterally restrains the beam top flange, while the moment connection between the purlins and the rafter top flange in conjunction with the full depth stiffeners provides torsion restraint.

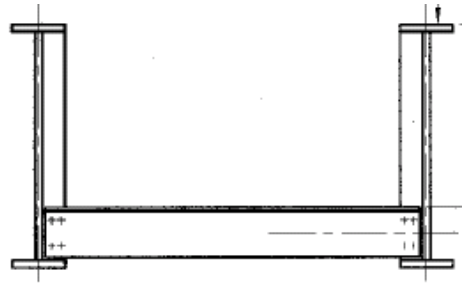

Figure 7: Main Beams Restrained by U Frames

The flexibility of the restraint system leads to the buckled profile shown in Figure 8.

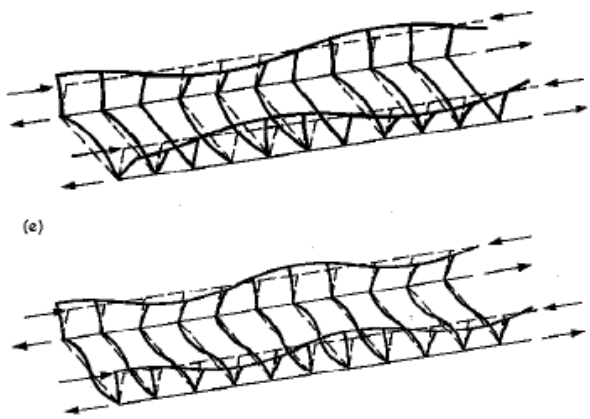

Figure 8: Buckled Shape of Compression Flange Restrained by U Frames (Jeffers, 1992)

The effective length for this type of structure is determined using the following formula (SAA, 2004).

$$L_e = 2.5k_1(EI_cL_u\Delta_u)^{0.25} \geq L_u$$

where

k_1 U-frame restraint effective length factor. This may be taken as 1.0. This is conservative where the compression flange is restrained against rotation in plan at the supports

I_c Second moment of area of the compression flange about its centroidal axis parallel to the web of the beam at the point of maximum moment

L_u Distance between frames

Δ_u Lateral deflection that would occur, in the U-frame, at the level of the centroid of the flange being considered, when a unit force acts laterally to the U-frame only at this point on the other flange or flanges connecting to the same frame. The direction of each unit load shall be such as to produce the maximum aggregate value of Δ_u. The U-frame shall be taken as fixed in position at each point of intersection of the deck and web, and as free and unconnected at all other points.

The value of Δ_u is made up of three components of deflection. In relation to the roof rafter situation there is deflection due to purlin rotation, deflection of the vertical stiffeners and flexibility of the purlin/ rafter moment connection. AS 5100.6 includes a formula for computing Δ_u. As an alternative a plane frame computer programme may be used to determine the maximum deflection under a unit load for the first two components of deflection. As there will typically be multiple rafters, several point loads should be considered. The direction of these unit loads should be such as to produce the worst bottom flange deflection.

The 2.5 factor in front of the effective length equation is related to the stiffness of the restraints at the end of the member. A value of 2.2 would apply for a fully rigid supports. The 2.5 factor is intended to account for some

support flexibility. If designers are uncertain as to the effectiveness of their supports, reference can be made to the British Standard BS 5400.3 Steel and Concrete Composite Bridges. In this publication the 2.5 factor is replaced by an equation which is a function of the torsional restraint provided by the supports. This factor varies from 2.22 to 3.6.

The U-frame effective length formula has been determined using beam on an elastic foundation theory. The strut is considered supported by a series of regularly spaced elastic springs (Figure 9). One of the implications of this approach is that the restraint system should be reasonably uniformly spaced.

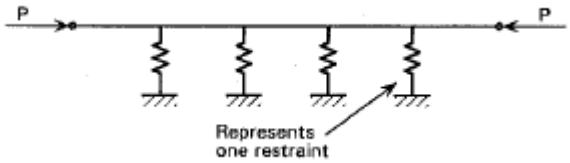

Figure 9: Plan View of Elastically Restrained Compression Flange (Brown and Iles, 2000)

Once the effective length has been computed the design proceeds as per the member bending provisions of the Steel Structures Standard.

Conclusions

Restraint system stiffness has an important role to play in the buckled shape, and therefore the moment causing instability of the critical compression flange of a beam. The purpose of this paper is to alert engineers to a particular situation where the current code provisions for the design of restraint systems which do not specify any minimum stiffness requirements may lead to un-conservative answers when computing the flexural capacity of deep rafters with flexible restraints. The Australian Bridge Standard (SAA, 2004) includes provisions for the design of structures where the point of application of the lateral restraint system is on the tension side of the member. Designers may use the U-frame provisions in this publication to compute an effective length used for buckling calculations that is based on the elastic stiffness of the tension side restraint system.

Flexibility of the restraining system of the restraining system is undesirable. To eliminate unnecessary restraint flexibility, rigid moment connections between the purlins and the rafter top flange should be used. For very deep rafter sections only a small rotation of the joint due to bolt slip can translate into significant displacement at the level of the bottom compression flange.

References

Brown, C., W. and Iles, D., C. (editors), Commentary on BS 5400-3:2000 Code of practice for the Design of Steel Bridges, Steel Construction Institute, Berkshire, 2000

BSI, Steel, Concrete and composite bridges – Part 3: Code of practice for design of steel bridges, British Standards Institute, London, October ,2000

Clifton, C., Restraint Classification for Beam Member Moment Capacity Determination to NZS 3404 1997, HERA report R4-92, New Zealand Heavy Engineering Research Association, Manukau City, 1997

Jeffers, E., Use of Braced Strut Behaviour to Assess BS 5400 Provisions for Beam Restraints, The Structural Engineer, Volume 70, No. 17, The Institution of Structural Engineers, London, September 1992

SAA, Bridge Design Part 6: Steel and composite construction, AS 5100.6, Standards Australia, Sydney, 2004

SAA, Steel Structures Standard, AS 1250, Standards Australia, Sydney 1981

SNZ, Steel Structures Standard, NZS3404 Part 1:and2 1997 (Incorporating Amendment 1 and 2), Standards New Zealand, Wellington, October 2007

Trahir, N., S., Bradford, M., A., The Behaviour and Design of Steel Structures, Chapman and Hall, London, 1988